

MA 425/525 first midterm review problems

Version as of September 11th.

The first midterm will be in class on Monday September 18th. No notes, books, or electronic devices will be allowed. Most of the exam will be closely based on problems, or on parts of problems, from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. If $5z^2 + 4z + 1 = 0$, then what is the real part of z ?
2. Let z and w be complex numbers. Simplify $|z|^2 - |z + \bar{w}|^2 + 2 \operatorname{Re}(wz)$.
3. Find all solutions to the equation $z^3 = 1 + i$ and sketch them in the complex plane.
4. Let C be the circle consisting of the points in the complex plane which are twice as far from $1 + i$ as they are from $1 - i$. Find the center and radius of C and sketch C in the complex plane.
5. For which values of the complex number a is $|az - 1| = |2iz + 3|$ the equation of a line?
6. For which values of the complex number a does the line $\operatorname{Im}(az) = 5$ cross the real axis at an integer point?
7. Find all points of continuity of the function

$$f(z) = \begin{cases} \frac{\pi z}{2i}, & |z| < 1 \\ \operatorname{Arg} z, & |z| \geq 1. \end{cases}$$

Sketch the set on which the function is discontinuous.

8. Let $f(z) = e^z$.
 - (a) Find all solutions to $f(iz) = i$ and sketch them in the complex plane.
 - (b) Find and sketch the range of f on $\{z: 0 < \operatorname{Im} z < \pi/2\}$.
 - (c) Find and sketch the largest possible domain on which f has the same range as in part (b).
9. Find all complex solutions to the following equations and sketch them in the complex plane:
 - (a) $e^{iz} = 2i$.
 - (b) $(z + 1)^i = 3$.
 - (c) $z^{2/3} = 1 + i$.
10. Parametrize the following curves and sketch them, indicating the orientation used: (a) $\{z: \operatorname{Re}((1 + i)z) = 5\}$, (b) $\{z: |z - 4 - 3i| = 2\}$.
11. Evaluate $\int_{\gamma} \bar{z} dz$, where γ is the curve in part (b) of problem 10.