MA 425/525 first midterm review problems
Version as of September 9th.

The first midterm will be in class on Monday September 16th. No notes, books, or electronic devices will be allowed. Most of the exam will be closely based on problems, or on parts of problems, from the list below. Justify your answers. Please let me know if you have a question or find a mistake.

1. If $5z^2 + 4z + 1 = 0$, then what is the real part of z?

2. Let z and w be complex numbers. Simplify $|z + w|^2 - |z - w|^2$.

3. Find all solutions to the equation $z^3 = 1 + i$ and sketch them in the complex plane.

4. Let C be the circle consisting of the points in the complex plane which are twice as far from $1 + 5i$ as they are from $1 - i$. Find the center and radius of C and sketch C in the complex plane. Do this problem once by using algebra to simplify $|z - p| = 2|z - q|$ for suitable p and q, and then again by computing the endpoints of the diameter of the circle on the line $\operatorname{Re} z = 1$.

5. For which values of the complex number a is $|az - 1| = |2iz + 3|$ the equation of a line?

6. For which values of the complex number a does the line $\operatorname{Im}(az) = 5$ cross the real axis at an integer point?

7. For which values of integer k and real number t does the series
$$\sum_{n=1}^{\infty} n^{2k} \Big(\frac{1 + it}{2}\Big)^n$$
converge? What about the series
$$\sum_{n=1}^{\infty} \frac{n^{2k}}{n!} \Big(\frac{1 + it}{2}\Big)^n?$$

8. For which z does $\sum_{n=3}^{\infty} (z + i)^n$ converge and what is the sum?

9. Find two ways to bring $|2iz - 1| = |4 - 3z|$ to the form $|z - p| = \rho|z - q|$ with $\rho > 0$ and p and q complex numbers, one with $\rho < 1$ and one with $\rho > 1$.

10. Find all complex solutions to the following equations and sketch them in the complex plane: (a) $e^{iz} = 2i$. (b) $(z + 1)^i = 3$. (c) $z^{2/3} = 1 + i$. (d) $z^{3/5} = 3i$.

11. Parametrize the following curves and sketch them, indicating the orientation used: (a) $\{z\colon \operatorname{Re}((1 + i)z) = 5\}$, (b) $\{z\colon |z - 4 - 3i| = 2\}$.

12. Evaluate $\int_\gamma \bar{z}dz$, where γ is the curve in part (b) above.

13. Let $\gamma = \{z\colon |2z + i|/|z - 1| = 2 \text{ and } \operatorname{Re} z \geq 0\}$. Parametrize γ and sketch it, indicating the orientation used. Then evaluate $\int_\gamma e^{-z}e^{-\bar{z}}dz$.